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It may also be observed that the theorem in this form is easily obtained from the generating function. For $(w:i,j)-(w-1:i,j)$ is the coefficient of $a^w x^j$ in the development of

$$\frac{1-a}{(1-x)(1-ax)(1-a^2x)\dots(1-a^ix)},$$

the numerator of which fraction may be written

$$1-a^m x-a\,(1-a^{m-1}x)\,,$$

so that $(w:i,j)-(w-1:i,j)$ is the difference between the coefficient of $a^w x^j$ in the development of

$$\frac{1-a^m x}{(1-x)(1-ax)\dots(1-a^ix)};$$

and the coefficient of $a^{w-1}x^j$ in the development of

$$\frac{1-a^{m-1}x}{(1-x)(1-ax)\dots(1-a^ix)};$$

and obviously if m is one of the numbers $1, 2, \dots i$, these coefficients are, respectively, the number of the (i,j) partitions of w which do not contain m, and the number of the (i,j) partitions of $w-1$ which do not contain $m-1$: wherefore, the partitions of w which do contain m and those of $w-1$ which do contain $m-1$ may be simultaneously rejected.

II.

Some General Formulæ for Integrals of Irrational Functions.

By W. I. Stringham, *Fellow of the Johns Hopkins University.*

Let $\mathrm{Sh}\,z=\frac{ax+c}{\sqrt{b^2-c^2}}$, $\mathrm{Ch}\,z=\frac{aX+c}{\sqrt{b^2-c^2}}$. Since $\mathrm{Ch}^2 z-\mathrm{Sh}^2 z=1$, therefore

$$aX+c=\sqrt{a^2x^2+2acx+b^2}\,.$$

Let $\frac{c}{a}=h$, and $\frac{b^2-c^2}{a^2}=e^2$. Then

$$\int(x+h)^m(X+h)^n\,dx=e^{m+n+1}\int\mathrm{Sh}^m z\ \mathrm{Ch}^{n+1}z\,.\,dz\,.$$

If the exponent of $(x+h)$ is an odd integer, writing $(\mathrm{Ch}^2 z - 1)^m \,\mathrm{Sh}\, z$ for $\mathrm{Sh}^{2m+1} z$ and developing by the binomial theorem, we find at once

$$(1)\qquad \int (x+h)^{2m+1} (X+h)^{2n-1}\, dx$$

$$= \sum_0^m{}_p\, (-e^2)^{m-p} \frac{m!}{p!\,(m-p)!} \cdot \frac{(X+h)^{2n+2p+1}}{2n+2p+1}.$$

If the exponent of $(x+h)$ is an even integer, by means of the usual reduction formulæ for $\mathrm{Sh}^{2m} z\ \mathrm{Ch}^{2m} z$, it will be found that

$$(2)\quad \int (x+h)^{2m} (X+h)^{2n-1}\, dx$$

$$= \frac{\left(n-\frac{1}{2}\right)!}{2\,(m+n)!} \sum_0^{n-1}{}_p\, e^{2p} \frac{(m+n-p-1)!}{\left(n-\frac{1}{2}-p\right)!} (x+h)^{2m+1} (X+h)^{2n-2p-1}$$

$$+ \frac{\left(m-\frac{1}{2}\right)!\ \left(n-\frac{1}{2}\right)!}{(m+n)!\ \left(\frac{1}{2}\right)!} \left(\sum_0^{m-1}{}_r\, (-)^r e^{2n+2r} \frac{(m-r-1)!}{\left(m-\frac{1}{2}-r\right)!} (x+h)^{2m-2r-1} (X+h) \right.$$

$$\left. + \frac{(-)^m}{\left(\frac{1}{2}\right)!} e^{2m+2n}\, \mathrm{Sh}^{-1} \frac{x+h}{e} \right)$$

Let $x+h-\mu = x+\lambda$. Then, by the binomial theorem,

$$(x+\lambda)^m (X+h)^n = \sum_0^m{}_p\, (-)^p \frac{m!\ \mu^p}{p!\ (m-p)!} e^{m+n-p}\, \mathrm{Sh}^{m-p} z\ \mathrm{Ch}^n z.$$

If the exponent of $(x+\lambda)$ is an even integer,

$$(3)\qquad \int (x+\lambda)^{2m} (X+h)^{2n-1}\, dx$$

$$= \sum_0^m{}_p\, \frac{(2m)!\ \mu^{2p}}{(2p)!\ (2m-2p)!} \frac{\left(n-\frac{1}{2}\right)!}{(m-p+n)!}$$

$$\times \left\{ \frac{1}{2} \sum_0^{n-1}{}_q\, \frac{(m-p+n-q-1)!}{\left(n-\frac{1}{2}-q\right)!} e^{2q} (x+h)^{2(m-p)+1} (X+h)^{2(n-q)+1} \right.$$

$$+\frac{\left(m-p-\frac{1}{2}\right)!}{\left(\frac{1}{2}\right)!}\left[\sum_{0}^{m-p-1}{}_{r}(-)^{r}\frac{(m-p-r-1)!}{\left(m-p-\frac{1}{2}-r\right)!}e^{2(n+r)}(x+h)^{2(m-p-r)-1}(X+h)\right.$$

$$\left.+\frac{(-)^{m-p}}{\left(\frac{1}{2}\right)!}e^{2(m-p+n)}\,\mathrm{Sh}^{-1}\frac{x+h}{e}\right]\Bigg\}$$

$$-\sum_{0}^{m}{}_{p}\frac{(2m)!\,\mu^{2p+1}}{(2p+1)!\,(2m-2p-1)!}\sum_{0}^{m-p-1}{}_{q}\left(-e^{2}\right)^{m-p-q-1}\frac{(m-p-1)!}{q!\,(m-p-q-1)!}\,\frac{(X+h)^{2(n+q)+1}}{2(n+q)+1}.$$

If the exponent of $(x+\lambda)$ is an odd integer,

$$(4)\quad \int(x+\lambda)^{2m+1}(X+h)^{2n-1}\,dx$$

$$=\sum_{0}^{m}{}_{p}\frac{(2m+1)!\,\mu^{2p}}{(2p)!\,(2m-2p+1)!}\sum_{0}^{m-p}{}_{q}\left(-e^{2}\right)^{m-p-q}\frac{(m-p)!}{q!\,(m-p-q)!}\,\frac{(X+h)^{2(n+q)+1}}{2(n+q)+1}$$

$$-\sum_{0}^{m}{}_{p}\frac{(2m+1)!\,\mu^{2p+1}}{(2p+1)!\,(2m-2p)!}\,\frac{\left(n-\frac{1}{2}\right)!}{(m-p+n)!}$$

$$\times\Bigg\{\frac{1}{2}\sum_{0}^{n-1}{}_{q}\frac{(m-p+n-q-1)!}{\left(n-\frac{1}{2}-q\right)!}e^{2q}(x+h)^{2(m-p)+1}(X+h)^{2(n-q)+1}$$

$$+\frac{\left(m-p-\frac{1}{2}\right)!}{\left(\frac{1}{2}\right)!}\left[\sum_{0}^{m-p-1}{}_{r}(-)^{r}\frac{(m-p-r-1)!}{\left(m-p-\frac{1}{2}-r\right)!}e^{2(n+r)}(x+h)^{2(m-p-r)-1}(X+h)\right.$$

$$\left.+\frac{(-)^{m-p}}{\left(\frac{1}{2}\right)!}e^{2(m-p+n)}\,\mathrm{Sh}^{-1}\frac{x+h}{e}\right]\Bigg\}.$$

When $\lambda=h$, μ vanishes, while for $p=0$, $\mu^{0}=1$, and the integrals (3) and (4) reduce to (2) and (1) respectively.

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